JOURNAL OF COMBINATORIAL THEORY, Series A 51, 314–315 (1989)

Note

Perfect One-Factorizations of K_{1332} and K_{6860}

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Communicated by the Managing Editors

Received January 1, 1988

We construct perfect 1-factorizations of K_{1332} and K_{6860} which are newly obtained.

The existence of a perfect 1-factorization of the complete graph K_{2n} for all $n \geqslant 2$ is conjectured and the problem is settled only for $2n = p+1$, $2p$ (p is prime), and $2n = 16$, 28, 36, 50, 244, 344 [4–7]. In this note, we construct perfect 1-factorizations of K_{1332} and K_{6860} by applying strong starters to $GF(p^m)$, $p^m \equiv 3 \pmod 4$.

Let p be a prime number and m be a natural number such that $p^m \equiv 3 \pmod 4$. We put $q = p^m$, $s = (q-1)/2$ and $2n = q+1$. $GF(q)$ denotes the Galois field with q elements and K_{2n} the complete graph with $2n$ vertices.

Let ω be a primitive element of $GF(q)$ and t be an odd integer such that $0 < t < 2s$. We define a starter 1-factor F_0:

$$F_0 = \{\{\omega^{2i}, \omega^{t+2i}\} \mid i = 0, 1, 2, ..., s-1\} \cup \{\{0, \infty\}\}.$$

For any $g \in GF(q)$, we put

$$F_g = F_0 + g = \{\{\omega^{2i} + g, \omega^{t+2i} + g\} \mid i = 0, 1, 2, ..., s-1\} \cup \{\{g, \infty\}\}.$$

Then

$$F(\omega^t) = \{F_g \mid g \in GF(q)\}$$

is a semi-regular 1-factorization of K_{2n}[2, 3].

0097-3165/89 $3.00

By suitable selections of the semi-regular 1-factorizations, we construct perfect 1-factorizations.

In case $p=11$ and $m=3$, let ω be a primitive element of $GF(11^3)$ with an minimal polynomial x^3+x^2+5. Then $F(\omega^7)$ is a perfect 1-factorization of K_{1332}.

In case $p=19$ and $m=3$, let ω be a primitive element of $GF(19^3)$ with a minimal polynomial x^3+x^2+16. Then $F(\omega^{127})$ is a perfect 1-factorization of K_{6860}.

These perfect 1-factorizations are newly obtained, to the best of the authors' knowledge.

Acknowledgments

The authors acknowledge Professor G. Nakamura, Dr. J. H. Dinitz, and the referees for their helpful advice.

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